MASSACHUSETTS
INSTITUTE of TECHNOLOGY
LIBRARIES

Digitized by the Internet Archive
in 2011 with funding from
MIT Libraries

http://www.archive.org/details/monopolisticcomp401blan

HB31
.M415
no.401

Dewey
MASS. INST. TECH.
DEC 4 1986
LIBRARIES

working paper
department
of economics

MONOPOLISTIC COMPETITION, AGGREGATE DEMAND
EXTERNALITIES AND REAL EFFECTS OF NOMINAL MONEY

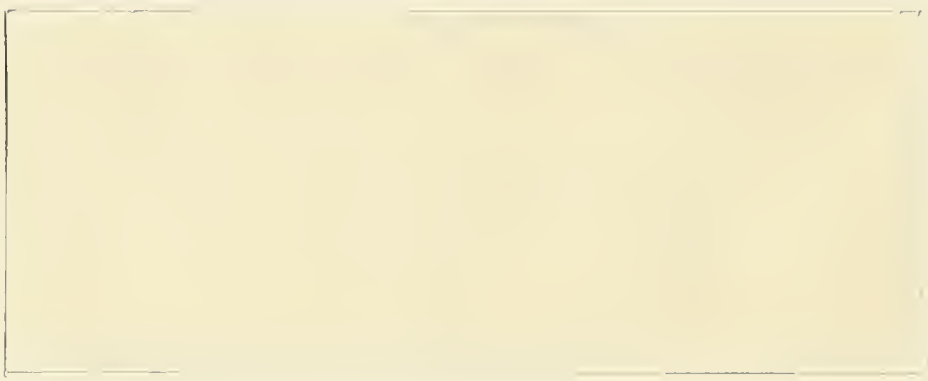
Olivier J. Blanchard

Nobuhiro Kiyotaki

November 198

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139



MONOPOLISTIC COMPETITION, AGGREGATE DEMAND EXTERNALITIES AND REAL EFFECTS OF NOMINAL MONEY

Olivier J. Blanchard

Nobuhiro Kiyotaki

No. 401 November 1985

M.I.T. LIBRARIES
DEC 3 1986
RECEIVED

Monopolistic competition, aggregate demand externalities and real effects of nominal money

Olivier J. Blanchard and Nobuhiro Kiyotaki

November 1985

* MIT and University of Wisconsin-Madison. We thank Andy Abel, Rudi Dornbusch, Bob Hall, Jeff Sachs, Larry Summers and Marty Weitzman for useful discussions.

ABSTRACT

A long standing issue in macroeconomics is that of the relation of imperfect competition to fluctuations in output. In this paper we examine the relation between monopolistic competition and the role of aggregate demand in the determination of output. We first show that monopolistically competitive economies exhibit an aggregate demand externality. We then show that, because of this externality, small menu costs, that is small costs of changing prices may lead to large effects of aggregate demand on output and on welfare.

A long standing issue in macroeconomics is that of the relation of imperfect competition to fluctuations in output. In this paper we examine the relation between monopolistic competition and the role of aggregate demand in the determination of output. We first show that monopolistically competitive economies exhibit an aggregate demand externality. We then show that, because of this externality, small menu costs, that is small cost of changing prices may lead to large effects of aggregate demand on output and on welfare.

The paper is organized as follows. Section I builds a simple general equilibrium model, with monopolistic competition in both labor and goods markets, and with nominal money ; it then characterizes the equilibrium. Section II characterizes the inefficiency associated with monopolistic competition and shows the inefficiency to be due to an aggregate demand externality. Section III studies the effects of changes in nominal money, when money is the numeraire, and when there are small, second order, costs of changing prices. It shows that changes in nominal money may have first order effects on output and welfare, and shows the close relation between this result and the results obtained in Section II.

Section I. A model of monopolistic competition

We want to construct a model in which each price setter is large in its own market but small with respect to the economy. The most convenient assumption is that of monopolistic competition. The simplest model of monopolistic competition, for our

purposes, would be one of households using labor to produce differentiated goods. However, because we want to focus later on both wage and price decisions, and want the model to be easily comparable to the standard macroeconomic model, we construct a model with both households and firms, and with separate labor and goods markets. Both labor and goods markets are monopolistically competitive. Each firm sells a product which is an imperfect substitute for other products ; each household sells a type of labor which is an imperfect substitute for other types. The assumption of monopolistic competition in both sets of markets is made for symmetry and transparence rather than for realism. Although we choose to interpret suppliers of labor as individual households, an alternative interpretation is to think of them as unions or syndicates (as in Hart (1982)).

The second choice follows from the need to avoid Say's law, or the result that the supply of goods produced by the monopolistically competitive firms automatically generates its own demand. To avoid this, we must allow agents to have the choice between consumption of these goods and something else. In the standard macroeconomic model, the choice is between consumption and savings. In other models of monopolistic competition, the choice is between produced goods and a non produced good (Hart, 1982 for example), or between produced goods and leisure (Startz 1985). Here, we shall assume that the choice is between buying goods and holding money. This is most simply and most crudely achieved by having real money balances in the utility function of agents. Thus, money plays the role of the non produced good and provides services[1,2].

---

[1] A Clower constraint would lead to similar results. Developing an explicitly intertemporal model just to justify why money is positively valued does not seem worth the additional complexity here.

[2] There are however differences between money and a non produced good, which arise from the fact that real, not nominal money balances enter utility ; we shall point out differences as we go along.

Money is also the numeraire, so that firms and workers quote prices and wages in terms of money ; this will play essentially no role in this and the next section, but will become important in Section III

The third choice is to make assumptions about utility and technology which lead to demand and pricing relations which are as close to traditional ones as possible, so as to allow an easy comparison with standard macroeconomic models. This however sometimes requires strong restrictions on utility and technology, which we shall indicate as we go along.

The model[3]

The economy is composed of m firms, each producing a specific good which is an imperfect substitute for the other goods, and n consumer- workers, households for short, each of them owning a type of labor which is an imperfect substitute for the other types. As a result, each firm has some monopoly power when it sets its price, and each worker has some monopoly power when he sets his wage[4]. We now describe the problem faced by each firm and each household.

Firms are indexed by i, i = 1,...,m. Each firm i has the following technology :

$$(1) \qquad Y_i = \left(\sum_{j=1}^{n} N_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{1}{\alpha}}$$

---

[3] The model can be viewed as an extension of the Dixit-Stiglitz (1977) model of monopolistic competition to macroeconomics.

[4] Since in equilibrium each labor supplier sells some of his labor to all firms, it is again more appropriate to think of labor suppliers as craft unions rather than individual workers. However since we want to analyze labor supply and consumption decisions simultaneously, we shall continue to refer to labor suppliers as "consumer-workers" or "households".

Y_i denotes the output of firm i. $N_{i,j}$ denotes the quantity of labor of type j used in the production of output i. There are n different types of labor, indexed j, j = 1,...,n. The production function is a CES production function, with all inputs entering symmetrically[5].

The two parameters characterising the technology are α and σ. The parameter σ is equal to the elasticity of substitution of inputs in production ; it will also be the elasticity of demand for each type of labor with respect to the relative wage. The parameter α is equal to the inverse of the degree of returns to scale ; α-1 will be the elasticity of marginal cost with respect to output -elasticity of marginal cost for short in what follows-. To guarantee the existence of an equilibrium, we limit ourselves to the case where σ is strictly greater than unity and where α is equal to or greater than unity.

Each period, the firm maximises profits. Nominal profits for firm i are given by :

$$(2) \qquad V_i = P_i Y_i - \sum_{j=1}^{n} W_j N_{i,j}$$

P_i denotes the nominal output price of firm i. W_j denotes the nominal wage associated with labor type j. The firm maximises (2) subject to the production function (1). It takes as given nominal wages and the prices of the other outputs. It also faces a downward sloping demand schedule for its product, which will be derived below as a result of utility maximisation by households. We assume that the number of firms is large enough that taking other prices as given is equivalent to taking the price level as given.

---

[5] We take the number of firms as given. The issue of whether there are fixed costs of production can therefore be left aside.

Households are indexed by j, j =1,...,n. Household j supplies labor of type j. It derives utility from leisure, consumption and real money balances. Its utility function is given by :

$$(3) \qquad U_j = (C_j)^{\gamma} (M_j'/P)^{1-\gamma} - N_j^{\beta}$$

$$\text{where} \qquad C_j = \left(\sum_{i=1}^{m} C_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

$$\text{and} \qquad P = \left(\frac{1}{m} \sum_{i=1}^{m} P_i^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

The first term, C_j, is a consumption index -basket- which gives the effect of the consumption of goods on utility. C_{ij} denotes the consumption of good i by household j. C_j is a CES function of the C_{ij}'s. All types of consumption goods enter utility symmetrically. The parameter θ is the elasticity of substitution between consumption goods in utility ; it will also be the elasticity of demand for each type of good with respect to its relative price. To guarantee existence of an equilibrium, θ is restricted to be greater than unity.

The second term gives the effect of real money balances on utility. γ is a parameter between zero and one. Nominal money balances are deflated by the nominal price index associated with C_j. We shall refer to P as the price level.

The third term in utility gives the disutility from work. N_j is the amount of labor supplied by household j. β-1 is the elasticity of marginal disutility of labor ; β is assumed to be equal to or greater than unity[67].

---

[67] The assumption that utility is homogeneous of degree one in consumption and real money balances, as well as additively separable in consumption and real money balances on the one hand and leisure on the other is made to eliminate income effects on labor supply. Under these assumptions, competitive labor supply would just be a function of the real wage, using the price index defined in the text. It also implies that utility is linear in income ; this facilitates welfare evaluations.

For reasons which will be clear below, we shall exclude the case where

Households maximise utility subject to a budget constraint. Each household takes prices and other wages as given. Again we assume that n is large enough that taking other wages as given is equivalent to taking the nominal wage level as given. It also faces a downward demand schedule for its type of labor, which will be derived as the result of profit maximisation by firms. The budget constraint is given by :

$$(4) \qquad \sum_{i=1}^{m} P_i C_{ij} + M_j' = W_j N_j + M_j + \sum_{i=1}^{m} V_{ij}$$

M_j denotes the initial endowment of money. V_{ij} is the share of profits of firm i going to household j.

The equilibrium

The derivation of the equilibrium is given in the appendix. The equilibrium can be characterised by a relation between real money balances and aggregate demand, a pair of demand functions for goods and labor and by a pair of price and wage rules :

The relation between real money balances and real aggregate consumption expenditures, which we shall call aggregate demand for short, is given by :

$$(5) \qquad Y = K\,(M/P) \qquad \text{where}$$

$$(6) \qquad Y \equiv \Big(\sum_{j=1}^{n}\sum_{i=1}^{m} P_i C_{ij}\Big)/P \quad \text{and} \quad P \equiv \Big((1/m)\sum_{i=1}^{m} P_i^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

The demand functions for goods and labor are given by :

---

both α and β are equal to unity.

$$(7) \qquad C_i = K_c \, (M/P) \, (P_i/P)^{-\theta} \qquad i=1,\ldots,m$$

$$(8) \qquad N_j = K_n \, (M/P)^{\alpha} \, (W_j/W)^{-\sigma} \qquad j=1,\ldots,m$$

where the wage index W is given by :

$$(9) \qquad W = \left(\frac{1}{n} \sum_{j=1}^{n} W_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

The price and wage rules are given by :

$$(10) \qquad (P_i/P) = [(\theta/(\theta-1))K_p \, (W/P) \, (M/P)^{\alpha-1}]^{(1/(1+\theta(\alpha-1)))} \qquad i=1,\ldots,m$$

$$(11) \qquad (W_j/W) = [(\sigma/(\sigma-1))K_w \, (P/W) \, (M/P)^{\alpha(\beta-1)}]^{(1/(1+\sigma(\beta-1)))} \qquad j=1,\ldots,n$$

The letters K, K_c, K_n, K_p, K_w are constants which depend on the parameters of the technology and the utility function as well as the number of firms and households.

We interpret these equations, starting with the relation between real money balances and aggregate demand. First order conditions for households imply a linear relation between desired real money balances and consumption expenditures. Aggregating over households and using the fact that, in equilibrium, desired money equal actual money gives equation (5).

The demand for each type of good relative to aggregate demand is a function of the ratio of its nominal price to the nominal price index, the price level, with elasticity $(-\theta)$. The demand for labor by firms is a derived demand for labor ; it depends on the demand for goods and thus on real money balances. The demand for each type of labor is a function of the ratio of its nominal wage to the nominal wage index, with elasticity $(-\sigma)$.

We now consider the price rule. Given the price level, each firm is a monopolist with non increasing returns to scale and decides about its real -or relative- price P_i/P. An increase in the real wage (W/P) shifts the marginal cost curve upward, leading to an increase in the relative price. An increase in real money balances shifts the demand curve for each product upward ; if the firm operates under strictly decreasing returns, the marginal cost curve is upward sloping and the relative price increases. If the firm operates under constant returns, the shift in aggregate demand has no effect on its relative price.

We finally consider the wage rule. We can think of households as solving their utility maximisation problem in two steps. They first solve for the allocation of their wealth, including labor income, between consumption of the different products and real money balances. After this step, the assumption that utility is linearly homogenous in consumption and real money balances implies that utility is linear in wealth, thus linear in labor income. The next step is to solve for the level of labor supply and the nominal wage. Given that utility is linear in labor income, we can think of households as monopolists maximising the surplus from supplying labor. Formally, if μ denotes the constant marginal utility of real wealth, households solve in the second step :

$$\max \quad \mu(W_j/P)\, N_j \; - \; N_j^{\beta} \qquad ; \quad N_j = K_n (M/P)^{\alpha} (W_j/W)^{-\sigma}$$

The real wage relevant for worker j is W_j/P, which we can write as the product $(W_j/W)(W/P)$. The demand for labor of type j is a function of the relative wage (W_j/W) as well as real money balances (M/P)

An increase in the aggregate real wage (W/P) leads household j to increase its labor supply, thus to decrease its relative wage (W_j/W). An increase in real money balances leads, if β is strictly greater than unity, to an increase in the relative wage. If β is equal to unity, if the marginal disutility of labor is constant, workers supply more labor at the same relative wage, in response to an increase in aggregate demand.

Symmetric equilibrium

Equilibrium and symmetry, both across firms and across households, implies that all relative prices and all relative wages must be equal to unity. Thus, using $P_i = P$ for all i and $W_j = W$ for all j, and substituting in equations (10) and (11) gives :

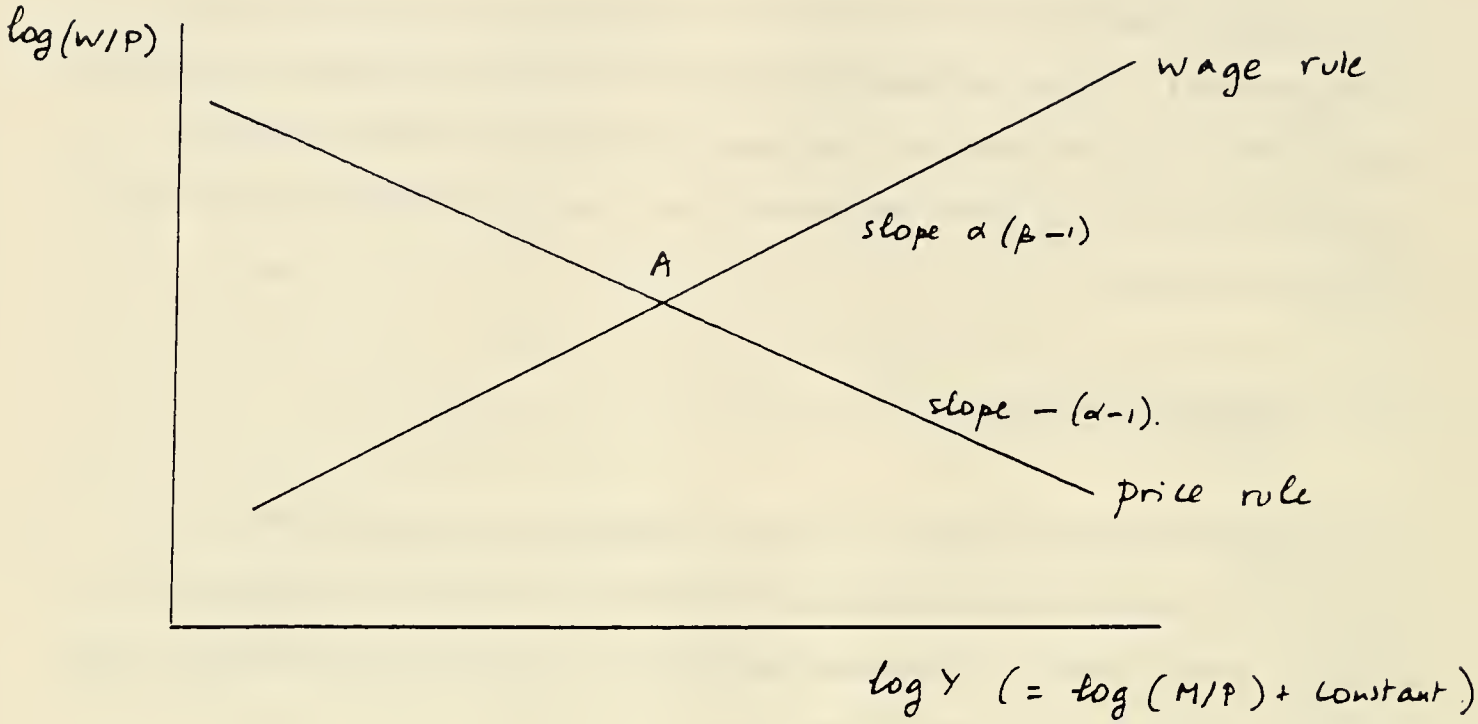
$$(12) \qquad (P/W) = (\theta/(\theta-1))\, K_p\, (M/P)^{\alpha-1}$$

$$(13) \qquad (W/P) = (\sigma/(\sigma-1))\, K_w\, (M/P)^{\alpha(\beta-1)}$$

Equation (12), which is obtained from the individual price rules and the requirement that all prices be the same gives the price wage ratio (P/W) as a function of real money balances. If firms operate under strictly decreasing returns, the price wage ratio is an increasing function of the level of output, thus of real money balances. Equivalently, the real wage (W/P) consistent with firms' behavior is a decreasing function of real money balances. We shall refer to equation (12) as the "aggregate price rule".

Equation (13), which is obtained from the individual wage rules and the requirement that all wages be the same gives the real wage (W/P) as a function of

FIGURE 1. THE MONOPOLISTICALLY COMPETITIVE EQUILIBRIUM.



real money balances. If β is strictly greater than unity, that is if workers have increasing marginal disutility of work, an increase in real money balances, which leads to an increase in the derived demand for labor, requires an increase in the real wage. The real wage consistent with households' behavior is an increasing function of real money balances. We shall refer to equation (13) as the "aggregate wage rule".

Equilibrium values of (W/P) and (M/P) are obtained from equations (12) and (13). The equilibrium value of output follows from (5). The equilibrium is characterised graphically in Figure 1. As (12) and (13) are log linear, we measure log(W/P) on the vertical axis and log(M/P) (or logY as the two are linearly related) on the horizontal axis. If α and β are both strictly greater than unity, the aggregate wage rule is upward sloping while the aggregate price rule is downward sloping. The equilibrium determines the real wage and real money balances. Given nominal money, it determines the price level. Given real money balances, we obtain the equilibrium level of aggregate demand and output.

Figure 1 looks very much like the characterization of equilibrium under perfect competition, with an upward labor supply curve and a downward sloping labor demand. What is therefore the effect of monopolistic competition ? This is the issue to which we now turn.

Section 2. Inefficiency and externalities

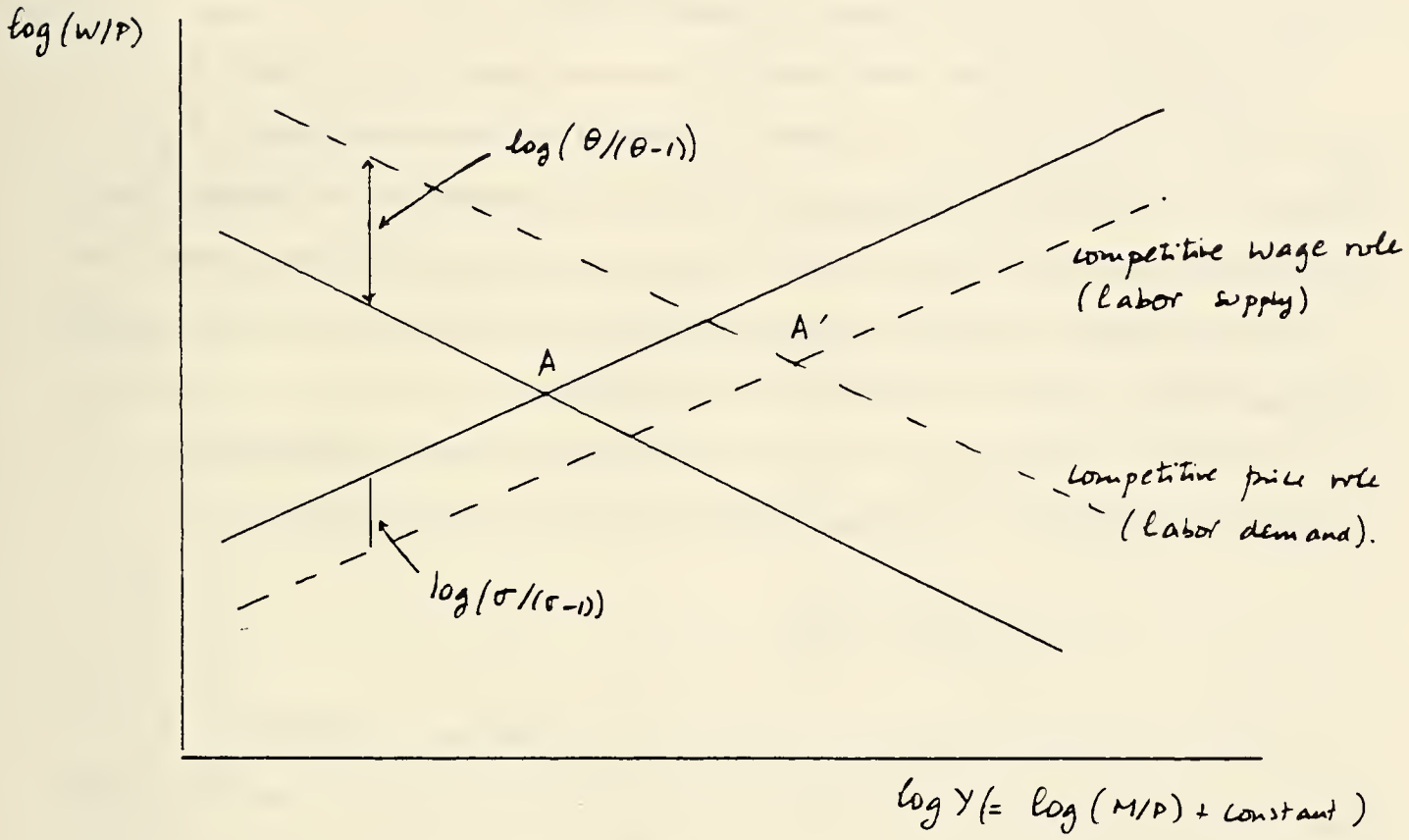
Comparing monopolistic competition and perfect competition

To characterize the inefficiency associated with monopolistic competition, we first compare the equilibrium to the competitive equilibrium. The competitive equilibrium is derived under the same assumptions about tastes, technology and the number of firms and households, but assuming that each firm (each household) takes its price (wage) as given when deciding about its output (labor).

The competitive equilibrium is very similar to the monopolistically competitive one. The demand functions for goods and labor are still given by equations (7) and (8). The price and wage rules are identical to equations (10) and (11), except for the absence of $\theta/(\theta-1)$ in the price rules and the absence of $\sigma/(\sigma-1)$ in the wage rules (the constant terms K_c, K_n, K_p, K_w and K are the same in both equilibria). (The derivation is left to the reader). The explanation is simple. The term $\theta/(\theta-1)$ is the excess of price over marginal cost, reflecting the degree of monopoly power of firms in the goods market ; if firms act competitively, price is instead equal to marginal cost. The same explanation applies to households.

Again, symmetry requires in equilibrium all nominal prices and all nominal wages to be the same ; this gives equations identical to (12) and (13), but without the terms $\theta/(\theta-1)$ in the aggregate price rule and $\sigma/(\sigma-1)$ in the aggregate wage rule. The price wage ratio consistent with firms' behavior is lower in the competitive case by $\theta/(\theta-1)$ at any level of real money balances (output); the real wage consistent with household's behavior is lower in the competitive case by $\sigma/(\sigma-1)$ at any level of real

FIGURE 2. MONOPOLISTICALLY COMPETITIVE AND COMPETITIVE EQUILIBRIA.



money balances. The monopolistically competitive and competitive aggregate wage and price rules are drawn in Figure 2. Point A' gives the competitive equilibrium, point A gives the monopolistically competitive equilibrium.

The equilibrium level of real money balances is lower in the monopolistic equilibrium ; the price level is higher. Employment and output are lower. What happens to the real wage is ambiguous and depends on the degrees of monopoly power in the goods and the labor markets. If, for example, there is monopolistic competition in the goods market but perfect competition in the labor market, then the real wage is unambiguously lower under monopolistic competition.

Denoting by R the ratio of output in the monopolistically competitive equilibrium to output in the competitive equilibrium, R is given by :

$$R = \left(\frac{\sigma-1}{\sigma} \; \frac{\theta-1}{\theta} \right)^{\frac{1}{\alpha\beta-1}} \qquad < 1$$

R is an increasing function of σ and θ. The higher the elasticity of substitution between goods or between types of labor, the closer is the economy to the competitive equilibrium. R is an increasing function of α and β. If α and β are both close to unity, R is small : the existence of monopoly power in either the goods or the labor markets can have a large effect on equilibrium output.

Aggregate demand externalities

Under monopolistic competition, output of monopolistically produced goods is too low. We have shown above that this follows from the existence of monopoly power in price and wage setting. An alternative way of thinking about it is that it follows from an aggregate demand externality.

The argument is as follows : in the monopolistically competitive equilibrium, each price (wage) setter has, given other prices, no incentive to decrease its own price (wage) and increase its output (labor). Suppose however that all price setters decrease their prices simultaneously ; this increases real money balances and aggregate demand. The increase in output reduces the initial distortion of underproduction and underemployment and increases social welfare[8].

We now make the argument more precise. By the definition of a monopolistically competitive equilibrium, no firm has an incentive to decrease its price, and no worker has an incentive to decrease its wage, given other prices and wages. Consider now a proportional decrease in all wages and all prices, $(dP_i/P_i) = (dW_i/W_j) < 0$, for all i and j, which leaves all relative prices unchanged but decreases the price level.

Consider first the change in the real value of firms[9]. At a given level of output and employment, the real value of each firm is unchanged. The decrease in the price level however increases real money balances and aggregate demand. This in turn shifts outward the demand curve faced by each firm and increases profit : an increase in demand at a given relative price increases profit as price exceeds marginal cost. Thus, the real value of each firm increases.

---

[8] An alternative way of stating the argument is as follows : If starting from the monopolistically competitive equilibrium, a firm decreased its price, this would lead to a small decrease in the price level and thus to a small increase in aggregate demand. While the other firms and households would benefit from this increase in aggregate demand, the original firm cannot capture these benefits and thus has no incentive to decrease its price. We have chosen to present the argument in the text to facilitate comparison with the argument of Section III.

[9] What happens to the real value of firms is obviously of no direct relevance for welfare. This step is however required to characterize what happens to the utility of households below.

Consider then the effect of a proportional reduction of prices and wages on the utility of each household. Consider household j. We have seen that, once the household has chosen the allocation of his wealth between real money balances and consumption, we can write its utility as:

$$U_j = \mu(I_j/P) - N_j^{\beta}$$

where μ is the constant marginal utility of real wealth and I_j is the total wealth of the j^{th} household. Using the budget constraint, we can express utility as :

$$U_j = [\mu (W_j/P)N_j - N_j^{\beta}] + \mu \sum_{i=1}^{m} V_{i,j}/P + \mu (M_j/P)$$

Utility is the sum of three terms. The second is profit income -in terms of utility- ; we have seen that each firm's profit goes up after an increase in aggregate demand. Thus, this term increases. The first term is the household's surplus from supplying labor. At a given level of employment, N_j, the proportional change in wages and prices leaves this term unchanged. But the increase in aggregate demand and the implied derived increase in employment implies that this term increases : at a given real wage, an outward shift in the demand for labor increases utility as the real wage initially exceeds the marginal utility of leisure. The third term is the real value of the money stock, which increases with the fall in the price level. Thus, utility unambiguously increases[10].

---

[10] Note that, if we were performing the same experiment in the neighborood not of the monopolistically competitive but of the competitive equilibrium, the first two terms would be equal to zero. The third one would however still oe present. This is one of the implications of our use of real money as the non produced good. If real money enters utility, then the competitive equilibrium is not a Pareto optimum, as a small decrease in the price level increases welfare. This inefficiency of the competitive equilibrium disappears if money is replaced by a non produced good, while the aggregate deamand externality under monopolistic competition remains valid (see Kiyotaki 1984).

The notion of an aggregate demand externality is an old idea in macroeconomics. It has been formalised in various recent papers; although these papers have on the surface relatively little in common, they share the following properties : an increase in one agent's activity increases the activity level and welfare of others ; a general increase in activity, if it can be engineered, by taxation or other means, may be welfare improving[11]. Diamond (1982) builds a macroeconomic model where trade takes place through search and shows that increased search by one trader has externalities as it increases the probability for other traders to find a profitable trade. Startz (1984) builds a macroeconomic model in which firms can not directly observe effort by individual workers. This leads to a payment scheme which has the implication that the optimal amount effort for each worker depends on the level of effort put in by other workers. In both cases, a small increase in activity is welfare improving.

Identifying the inefficiency associated with monopolistic competition as an aggregate demand externality does not however imply that movements in aggregate demand affect output. Consider for example changes in nominal money[12]. As equations (12) and (13) are homogeneous of degree zero in P, W and M, nominal money is obviously neutral, affecting all nominal prices and wages proportionately and leaving output and employment unchanged[13]. Thus something else is needed to obtain real

---

[11] A similar point is made by Cooper and John [1985].

[12] As we have not specified how money is introduced in this economy, it is best to think of them as helicopter drops.

[13] Here, and in the next section, instead of focusing on the effects of aggregate demand on output in general, we focus for convenience on the more narrow question of whether changes in nominal money have real effects. The results here and in the next section would apply equally to non monetary pure aggregate demand shifts, i.e. shifts which leave labor supply unchanged at a given real wage, where the real wage is defined as the wage in terms of the consumption basket. If we modify the utility function to be

effects of nominal money. We examine the effects of costs of price setting in the next section.

$$U_j = (C_j)^{\gamma}((M_j/P)+\varepsilon)^{1-\gamma} - N_j^{\beta} - \mu\,\varepsilon$$

Then shifts in ε will shift the demand for goods given real money balances, while leaving labor supply unchanged at a given real wage and are therefore pure aggregate demand shocks. By contrast, shifts in γ are not pure aggregate deamnd shocks.

Section 3. Menu costs and real effects of nominal money

We now introduce small costs of setting prices, small "menu" costs. There are obviously costs to changing prices, which range from the cost of changing tags and printing new catalogs to gathering the information needed to choose the new prices, informing new customers of these prices and so on. The question however is whether these costs, which cannot be very large, can have important macroeconomic effects. This section shows that they may. Small menu costs may imply large movements in activity in response to demand, and may have large welfare effects[14].

The first part of the section formalizes the argument for small changes in nominal money, and shows the close relation between the aggregate demand externality argument of the previous section and the argument presented in this section. The second part considers larger changes in nominal money, and focuses on the effects of structural parameters on the ratio of output and welfare effects to menu costs.

---

[14] We are not the first to make this point. Mankiw (1985) has pointed out that, under imperfect competition, private and social costs of price setting could differ substantially, leaving open the possibility of large welfare effects of demand changes. Akerlof and Yellen (1985a, 1985b) have emphasized the potential welfare effects of near rationality under imperfect competition. Decision makers are said to be "near rational" if they react to changes in the environment only if not reacting would entail a first order loss. As Akerlof and Yellen point out however, near rationality can be described as full rationality subject to second order costs of taking decisions, so that their analysis is directly relevant to this section. Our contribution is to point out the relation to the aggregate demand externality emphasized in the last section, and because our model is more explicitly based on utility and profit maximisation, to give a more detailed welfare analysis.

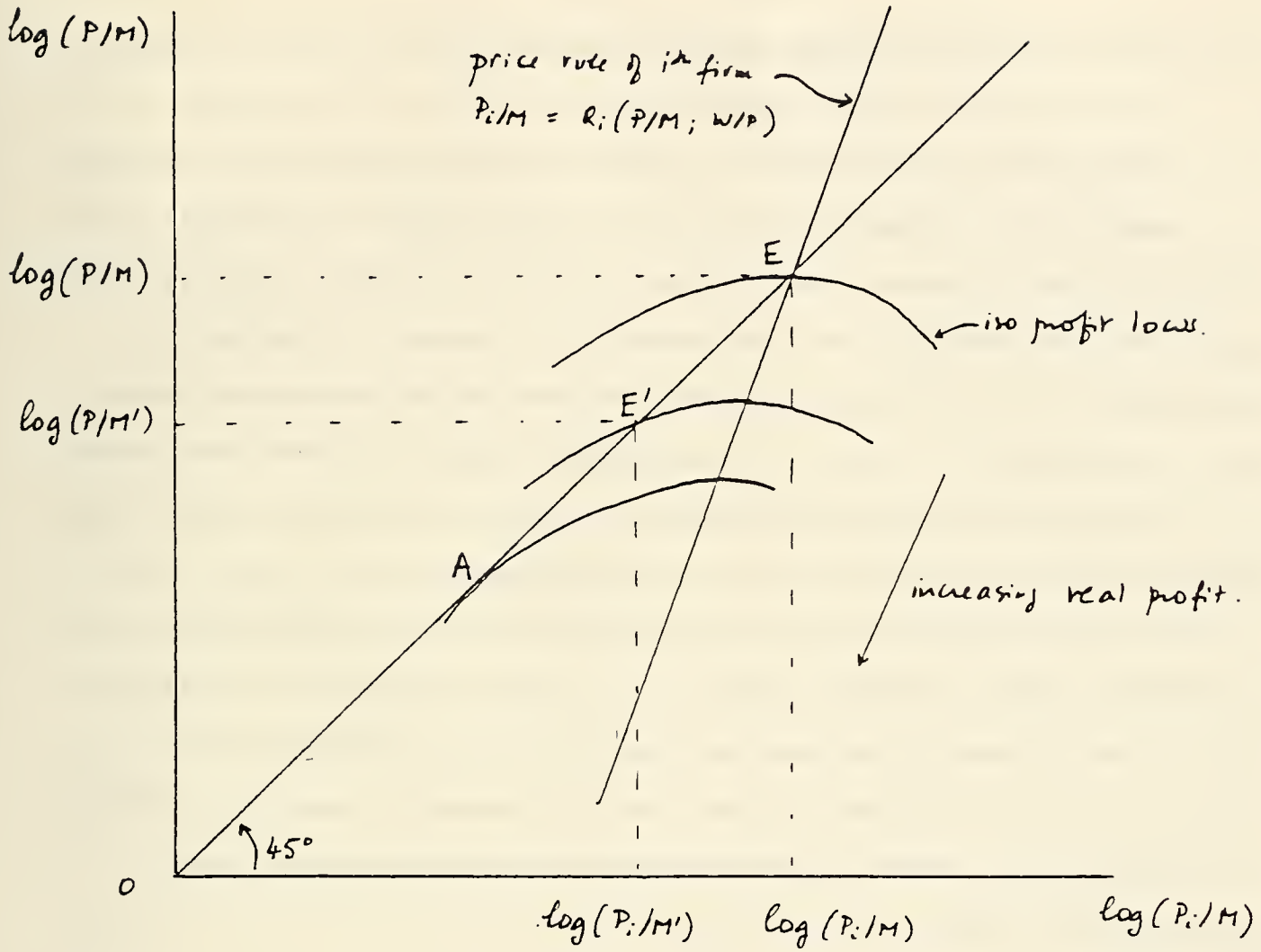
The effects of small changes in nominal money

We start by considering the effects of a small change in nominal money, dM, starting from the equilibrium described in the first section. The argument proceeds as follows :

At the initial nominal prices and wages, the change in nominal money leads to a change in aggregate demand, thus to a change in the demand facing each firm. If demand is satisfied, the change in output implies in turn a change in the derived demand for labor, thus a change in the demand facing each worker. Unless firms operate under constant returns, each firm wants to change its relative price. Unless workers have constant marginal utility of leisure, each worker wants to change his relative wage. We show however that the loss in value to a firm which does not adjust its relative price is of second order ; the same is true of the utility of a worker who does not adjust his relative wage. Thus second order menu costs may prevent firms and workers from adjusting prices and wages. The implication is that nominal prices and nominal wages do not adjust to the change in nominal money. The second part of the argument is to show that the change in real money balances has first order effects on welfare ; we show that the effect on welfare is indeed first order, and of the same sign as the change in money. The argument has very much the same structure as the aggregate demand externality argument of the previous section ; this coincidence is not accidental and we return to it below.

The first part is a direct application of the envelope theorem. Consider firms first. Let V_i be the value of firm i. V_i is a function of P_i as well as of P, W and M : $V_i = V_i(P_i,P,W,M)$. Let V_i^* be the maximised value of firm i, after maximisation over P_i : $V_i^* = V_i^*(P,W,M)$. The envelope theorem then says that :

FIGURE 3. AGGREGATE DEMAND EXTERNALITIES AND MENU COSTS



$$dV_i^*/dM = \partial V_i/\partial M + (\partial V_i/\partial P_i)(dP_i/dM) = \partial V_i/\partial M$$

To a first order, the effect of a change in M on the value of the firm is the same whether or not it adjusts its price optimally in response to the change in M. Exactly the same argument applies to the utility of the household. Thus, second order menu costs (but larger than the second order loss in utility or in value) will prevent each firm from changing its price given other prices and wages and each worker from changing its wage given other prices and wages. The implication is that all nominal prices and wages remain unchanged and that the increase in nominal money implies a proportional increase in real money balances.

What remains to be shown is that the change in real money balances has positive first order effects on welfare. However, as we have already shown in the previous section, the increase in real money balances, associated with the increase in aggregate demand and employment, raises firms'profits and the households' surpluses from supplying labor. Thus, it increases welfare in the neighborood of the monopolistically competitive equilibrium.

The relation between aggregate demand externalities and the argument of this section is illustrated using the diagram in Figure 3[15].

Figure 3 plots the price rule (10) giving the price chosen by firm i as a function of the price level. The logarithm of the price level is on the vertical axis while the logarithm of the price of the i^{th} firm is on the horizontal one ; both are

---

[15] The reason why the argument below is only an illustration is that it only looks at firms, taking the real wage as given ; it is thus only a partial equilibrium argument. The argument would be a general equilibrium one if we were looking at an economy composed of households, each producing a differentiated good.

measured as ratios to nominal money. The price rule is drawn for a given real wage (W/P) (assumed to be set at its monopolistically competitive value) and gives $\log(P_i/M)$ as a linear function of $\log(P/M)$. In the presence of monopoly power, the price rule has slope greater than one. We also draw isoprofit loci, giving combinations of (P_i/M) and (P/M) which yield the same level of real profit for the firm[16]. The symmetric monopolistically competitive equilibrium is given by the intersection of the price rule and the 45 degree line, point E. Point A gives the highest real profit point on the 45 degree line.

The aggregate demand externality argument can then be stated as follows. Consider a small proportional decrease of prices, keeping nominal money and the real wage constant. The equilibrium moves from point E to a point like E' along the 45 degree line. The profit of each firm rises with the increase in aggregate demand. However, in the absence of coordination, no firm has an incentive to reduce prices away from the equilibrium point E.

The menu cost argument considers instead a small increase in nominal money. At the initial set of prices, real money balances would increase and the economy would move from point E to a point like point E'. But, absent menu costs, each firm would find in its interest to increase its price until the economy had returned to point E. In the presence of menu costs however, these menu costs, if large enough, can prevent this movement back to E, so that the economy remains at E' and all firms end up with higher real profits.

---

[16] The figure assumes decreasing returns to scale. Note also that, as firms take the price level as given when choosing their own price, isoprofit loci are horizontal along the price rule.

A similar argument, although slightly more complicated, holds for wages. We shall not present it here.

It is also important to note the specific role played by money in this section. The presence of an aggregate demand externality does not depend on the nature of the produced good, and on the nature of the numeraire. The results of this section depend on money being the non produced good and the numeraire. That money is the numeraire implies that, given menu costs, unchanged prices and wages mean unchanged nominal prices and wages. That money is the non produced good implies that as the government can vary the amount of nominal money, it can, if nominal prices and wages do not adjust, change the amount of real money balances, the real quantity of the non produced good.

The effects of larger changes in nominal money

If we want to examine the effects of larger changes in nominal money, we can no longer use the result derived above, for its proof relies on the assumption of small changes in money. For larger changes, the private opportunity costs of not adjusting prices in response to the change in money -private costs, for short- are no longer negligible and depend on the parameters of the model. We now investigate this dependence.

The private costs faced by a firm depends on the size of the demand shifts as well as on the two parameters α and θ. As we have seen, these costs are of second order in response to a change in aggregate demand, thus roughly proportional to the square of the change in aggregate demand. More precisely, define $L(\Delta\ ;\ \alpha,\theta)$ to be the private opportunity cost to a firm expressed as a proportion of initial revenues, associated with not adjusting its price in response to a change of $100\Delta\%$ in aggregate

demand, when all other firms and households keep their prices and wages unchanged. Then, by simple computation, we get :

$$L(\Delta\ ;\ \alpha,\theta) = \{[(\alpha-1)^2(\theta-1)]/[2(1+\theta(\alpha-1))]\}\ \Delta^2 + o(\Delta^2)$$

where $o(\Delta^2)$ is of third order.

The closer α is to one, i.e the closer to constant returns are the returns to scale, the smaller the private cost. In the limit, if α is equal to one, then private costs of not adjusting prices are equal to zero as the optimal response of a monopolist to a multiplicative shift in isoelastic demand under constant marginal cost is to leave the price unchanged. Thus private costs are an increasing function of α. They are also an increasing function of θ ; the higher the elasticity of demand with respect to price, the higher the private costs of not adjusting prices.

Exactly the same analysis applies to workers. The two important parameters for them are β and σ. If we define the function L in the same way as above, the private opportunity cost to a worker, measured in terms of consumption and expressed as a proportion of initial consumption), associated with not adjusting the wage in response to a change of $100\Delta\%$ in aggregate demand, when all other firms and households keep their prices and wages unchanged, is given by :

$$[(\theta-1)/\theta\alpha]\ L(\ (1+\Delta)^{\alpha} -1\ ;\ \beta,\sigma),$$

where $(\theta-1)/\theta\alpha$ is the initial share of wage income in GNP.

If β is close to unity, i.e if the elasticity of the marginal disutility of labor is close to unity, private costs of not adjusting wages are small ; in the limit, if marginal disutility of labor is constant, private costs are equal to zero. If σ is very large, if labor types are close substitutes, private costs of not adjusting wages are high.

Table 1a gives the size of menu costs as a proportion of the firm's revenues (GNP produced by the firm) which are just sufficient to prevent a firm from adjusting its price in response to a change in demand ; Table 1b gives the size of menu costs (in terms of consumption) as a proportion of initial consumption (GNP consumed by the worker) which are just sufficient to prevent a worker from adjusting his wage.

Table 1 Changes in aggregate demand and menu costs

(a) Loss in value to a firm from not adjusting prices (as a proportion of initial revenues)				(b) Loss in utility (in terms of consumption) to a worker from not adjusting wages (as a proportion of initial consumption)*			
		M_1/M_0 =				M_1/M_0 =	
alpha	theta	1.05	1.10	beta	sigma	1.05	1.10
1.1	5	.003%	.013	1.4	5	.066%	.265
1.1	2	.001	.004	1.4	2	.027	.111
	20	.008	.031		20	.105	.418
1.0	5	.000	.000	1.2	5	.025	.100
1.3	5	.018	.071	1.6	5	.112	.451

* : $\theta = 5$; $\alpha = 1.1$

M_0 is the initial level of nominal money, M_1 the level after the change.

Thus, given the unit elasticity of aggregate demand with respect to real money balances and the assumption that all other prices have not changed, table 1a gives the private costs associated with not changing prices in the face of 5% and 10% changes in demand to the firm. The main conclusion is that very small menu costs ,say less than .01% of revenues, may be sufficient to prevent adjustment of prices. Results are qualitatively similar for workers. Table 1b gives the private costs of not changing the effects of changes of +5% and +10% in the demand for final goods. It

assumes that α is equal to 1.1, so that changes in the derived demand for labor are of 5.5% and 11% approximately. We expect β to be higher than α so that table 1b looks at values of β between 1.2 and 1.6. For values of β close to unity, required menu costs are again very small ; as β increases however, required menu costs become non negligible : for β=1.6 and a 11% change in demand, they reach .45% of initial consumption, a number which is no longer negligible.

The more relevant comparison however, at least from the point of view of welfare, is between private costs and welfare effects, i.e. the change in utility resulting from the changes in output, employment and real money which are implied by a change in nominal money at given prices and wages. Welfare effects depend on the size of the change in nominal money as well as on the parameters α, β, θ and σ ; the dependence is a complex one and we shall not analyze it here in detail. Table 2 gives numerical examples. It gives the required menu costs and welfare effects associated with two different changes in nominal money, 5% and 10% and different values of the structural parameters.

For each of the two changes in money, the first column gives the minimum value of menu costs, expressed as a proportion of GNP, which prevents adjustment of nominal prices and wages ; this value is the sum of menu costs required to prevent firms from adjusting their prices and workers from adjusting their wages, given other wages and prices. The second column gives the welfare effects of an increase in nominal money at unchanged prices and wages, expressed in terms of consumption, again as a proportion of GNP. The third gives the ratio of welfare effects to menu costs.

Table 2 Menu costs and welfare effects

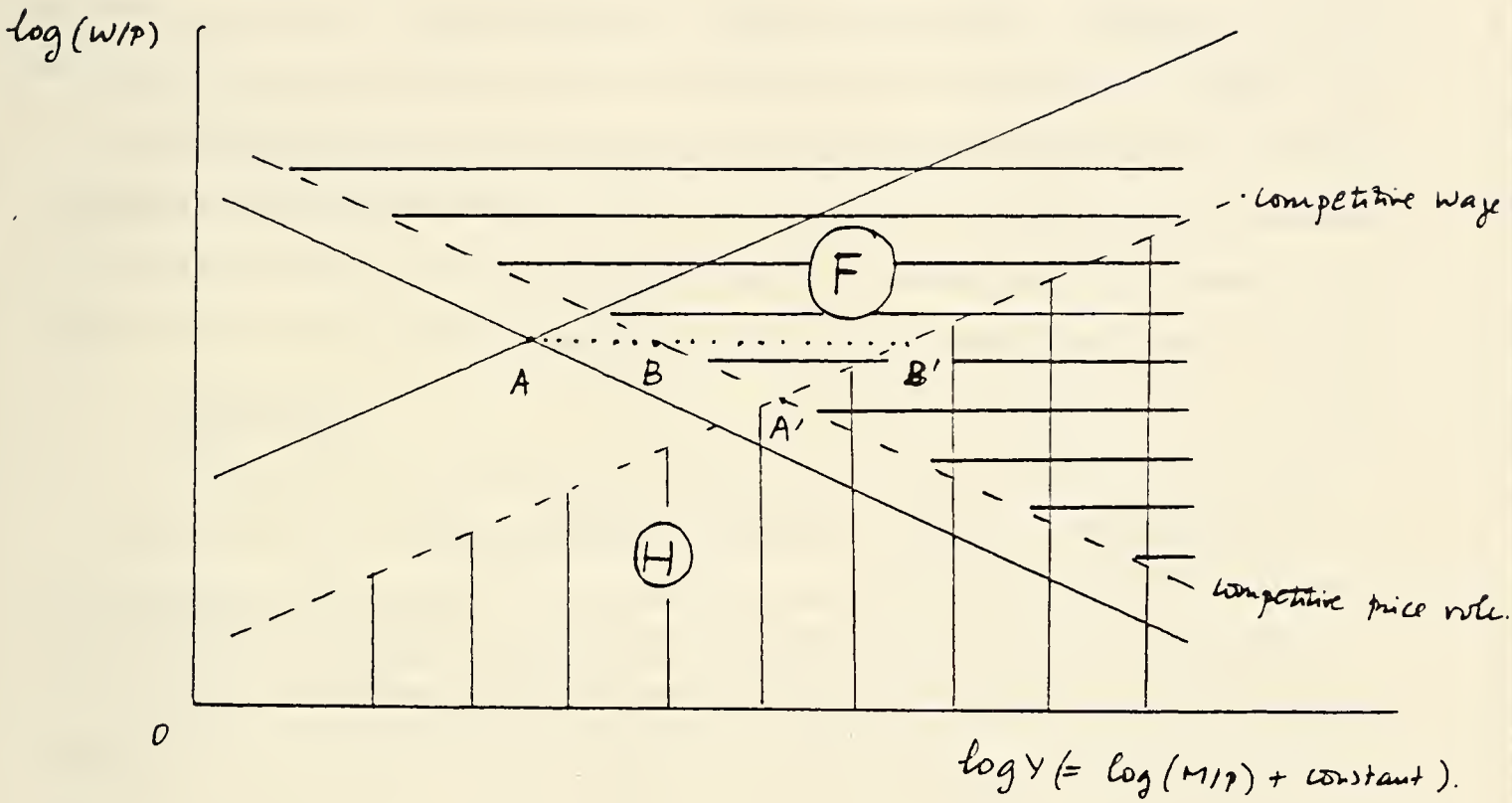
		$M_1/M_0 = 1.05$			$M_1/M_0 = 1.10$		
alpha	beta	Menu Costs	Welfare Effects	Ratio	Menu Costs	Welfare effects	Ratio
($\theta=\sigma=5$)							
1.1	1.2	.03%	1.79%	60	.11%	3.54%	32
	1.4	.07%	1.83%	26	.28%	3.60%	13
	1.6	.11%	1.91%	17	.46%	3.72%	8
1.2	1.2	.04%	1.82%	45	.15%	3.57%	24
	1.4	.08%	1.87%	24	.33%	3.67%	11
	1.6	.13%	1.98%	15	.53%	3.85%	7
($\theta=\sigma=10$)							
1.1	1.2	.03%	.94%	31	.11%	1.86%	17
	1.4	.06%	1.02%	17	.23%	1.93%	8
	1.6	.09%	1.11%	12	.36%	2.05%	6
1.2	1.2	.04%	.99%	25	.16%	1.87%	12
	1.4	.07%	1.07%	16	.29%	2.01%	7
	1.6	.11%	1.27%	12	.44%	2.24%	5

Welfare effects turn out not to be much affected by the specific values of the parameters, at least for the range of values we consider in the table. Thus, the ratio of welfare effects to menu cost has the same qualitative behavior as that of the ratio of output movements to menu costs. It is largest for values of α, β, θ and σ close to unity, and decreases as these parameters increase. In the table, it varies from 60 for low values of α, β, θ and σ to 5 for high values of these parameters.

Demand determination of output

We have until now assumed that increases in real money balances at constant prices and wages led to increases in output and employment. When we were analyzing the effects of small changes in money, this assumption was clearly warranted ; in the

FIGURE 4. DEMAND DETERMINATION OF OUTPUT.



initial monopolistically competitive equilibrium, as price exceeds marginal cost, firms will always be willing to satisfy a small increase in demand at the existing price. The same is true of workers : as the real wage initially exceeds the marginal disutility of labor, workers will willingly accomodate a small increase in demand for their type of labor. When we consider larger changes in money, this may no longer be the case. Even if firms do not adjust their price, they have the option of either accomodating or rationing demand ; they will resort to the second option if marginal cost exceeds price. The same analysis applies to workers. From standard monopoly theory, we know that firms and workers will accomodate relative increases in demand of :

$$(\theta/(\theta-1))^{\frac{1}{\alpha-1}} \qquad \text{and} \qquad (\sigma/(\sigma-1))^{\frac{1}{\beta-1}} \qquad \text{respectively}$$

This raises the question of whether, assuming menu costs to be large enough, an increase in demand can increase output all the way to its competitive level. The answer is provided in Figure 4. Figure 4 replicates Figure 2 and draws the aggregate price and wage rules under competitive and monopolistically competitive conditions. A is the monopolistic competitive equilibrium, A' the competitive one. Along the monopolistically competitive price rule, price exceeds marginal cost ; thus firms will satisfy demand, at a given price wage ratio, until marginal cost equals price, that is until they reach the competitive locus. In our case, firms will supply up to point B. The shaded area F is the set of output-real wage at which firms will ration rather than supply. By a similar argument, workers will supply up to point B'. The shaded area H is the set of real wage combinations where workers do not satisfy labor demand. The figure makes it clear that an increase in nominal money will increase output and employment. It also makes clear that, no matter how large menu costs are,

it is impossible, unless the competitive and monopolistically competitive real wages are equal, to attain the competitive equilibrium through an increase in nominal money.

What happens therefore as demand increases depends on both menu costs and supply constraints. If menu costs are large, supply constraints will come into effect first. If menu costs are small, a more likely case, prices and wages adjust before supply constraints come into effect.

Conclusion

The results of this paper are tantalizingly close to those of traditional Keynesian models : under monopolistic competition, output is too low, because of an aggregate demand externality. This externality, together with small menu costs, implies that movements in demand can affect output and welfare. In particular, increases in nominal money can increase both output and welfare. In fact, while we believe these results to be important to the understanding of macroeconomic fluctuations, it is also clear that there is still a long way to go for this model to justify Keynesian results. Let us mention some of the main issues.

The scope for small menu costs to lead to large output, employment and welfare effects in our model depends critically on the elasticity of labor supply with respect to the real wage being large enough (on $(\beta-1)$ being small). Evidence on individual labor supply suggests however a small elasticity. Thus the "menu cost" approach runs into the same problem as the imperfect information approach to output fluctuations : neither can easily generate large fluctuations in output in response to demand if the real wage elasticity of labor supply is low. As in the imperfect information case, the theory may be rescued by the distinction between temporary and

permanent changes in demand. An other possibility is that unions have a flatter labor supply than individuals. More likely, the assumption that labor markets operate as spot markets (competitive or monopolistically competitive) may have to be abandoned[17].

The analysis of this paper is purely static. There are substantial conceptual issues in extending the model to look at the dynamic effects of demand on output, in the presence of menu costs. If menu costs lead to staggered nominal price and wage decisions, with fixed lengths of time between decisions, the model delivers, depending on the particular staggering structure, the same qualitative results as recent macroeconomic models with staggering, such as those by Akerlof (1969), Taylor (1979) and Blanchard (1983) (see Blanchard (1985) for a more detailed argument). If however menu costs lead price and wage setters to use (S,s) policies, which imply random periods of time between decisions, the results may be quite different ; in response to a change in aggregate demand, only a few prices may be readjusted ; they may however be readjusted by a large amount, implying a large change in the price level, and little effect of real money on output, apart form the distortions on the price structure (see Caplin and Spulber(1985), and Blanchard and Fischer (1985) for further discussion).

---

[17] This is the direction taken by Akerlof and Yellen (1985b) who formalize the goods market as monopolistically competitive and the labor market using the "efficiency wage" hypothesis.

Appendix

This appendix derives the market equilibrium conditions (5) to (11) given in the text and proceeds in three steps. The first derives the demand functions of each type of labor and each type of product by solving part of the maximization problems of firms and households. These functions hold whether or not prices and wages are set by workers and firms at their profit or utility maximizing level. The second derives price rules from firms' profit maximization and wage rules from workers' utility maximization. The third characterises market equilibrium.

1. Demands for product and labor types

a) In order to maximize profit, each firm minimizes its production cost for a given level of output and wages :

$$\min_{N_{i,j}} \sum_{j=1}^{n} W_j N_{i,j} \quad \text{subject to} \quad \left(\sum_{j=1}^{n} N_{i,j}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{1}{\alpha}} \geq Y_i$$

Solving this minimisation problem gives :

$$N_{i,j} = (n^{\frac{\sigma}{1-\sigma}}) \, (W_j/W)^{-\sigma} \, Y_i^{\alpha}$$

$$\text{and} \quad \sum_{j=1}^{n} W_j N_{i,j} = (n^{\frac{1}{1-\sigma}}) \, W \, Y_i^{\alpha} \tag{a1}$$

$$\text{where} \quad W = \left((1/n) \sum_{j=1}^{n} W_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{a2}$$

The demand for labor of type j is therefore given by :

$$N_j = \sum_{i=1}^{m} N_{i,j} = (W_j/W)^{-\sigma} \, N/n \tag{a3}$$

$$\text{where} \quad N \equiv \left(\sum_{i}^{m} \sum_{j}^{n} W_j N_{i,j} \right)/W = (n^{\frac{1}{1-\sigma}}) \sum_{i=1}^{m} Y_i^{\alpha} \tag{a4}$$

N can be interpreted as the aggregate labor index

b) In order to maximize utility, each household chooses the optimum composition of consumption and money holdings for a given level of total wealth I_j and product prices :

$$\max_{C_{ij},M_j'} \Lambda_j = \left(\sum_{i=1}^{n} C_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta\gamma}{\theta-1}} (M_j'/P)^{1-\gamma}$$

$$\text{subject to} \quad \sum_{i=1}^{n} P_i C_{ij} + M_j' = I_j$$

Solving this maximization problem gives :

$$C_{ij} = (P_i/P)^{-\theta} (\gamma I_j/Pm) \tag{a5}$$

$$M_j' = (1-\gamma) I_j \quad \text{and} \tag{a6}$$

$$\Lambda_j = \mu I_j/P \tag{a7}$$

$$\text{where } P = \left((1/m) \sum_{i=1}^{m} P_i^{1-\theta}\right)^{\frac{1}{1-\theta}} \quad \text{and} \quad \mu = (\gamma m)^{\frac{1}{\theta-1}} (1-\gamma)^{1-\gamma} \tag{a8}$$

μ can be interpreted as the marginal utility of real wealth

The <u>demand for product of type i</u> is therefore given by :

$$Y_i = \sum_{j=1}^{n} C_{ij} = (P_i/P)^{-\theta} (Y/m) \tag{a9}$$

$$\text{where } Y = \left(\sum_{j}^{n} \sum_{i}^{m} P_i C_{ij}\right)/P = (\gamma/P) \sum_{j=1}^{n} I_j \tag{a10}$$

Y denotes real aggregate consumption expenditures of households and will be referred to as "aggregate demand".

Note that (a5), (a6), (a9) and (a10) imply the following <u>relation between aggregate demand and aggregate desired real money balances</u> :

$$Y = (\gamma/(1-\gamma)) M'/P \quad \text{where} \quad M' = \sum_{j=1}^{n} M'_j \tag{a11}$$

<u>2. Price and wage rules</u>

a) Taking as given wages and the price level, each firm chooses its price and output so as to maximize profit :

$$V_i = P_i Y_i - \sum_{j=1}^{n} W_j N_{ij} \tag{a12}$$

subject to the cost function (a1) and the demand function for its product (a9). Solving the above maximization problem gives :

$$P_i = (\theta/(\theta-1)) n^{\frac{1}{1-\sigma}} \alpha Y_i^{\alpha-1} W, \text{ or equivalently} \tag{a13}$$

$$P_i/P = [((\theta/(\theta-1)) n^{\frac{1}{1-\sigma}} m^{1-\alpha})(W/P)(Y^{\alpha-1})]^{\frac{1}{(1+\theta(\alpha-1))}} \tag{a14}$$

Equation (a11) implies that the price is equal to $\theta/(\theta-1)$ times the marginal cost.

b) Taking as given prices and other wages, each household chooses its wage and labor supply so as to maximize utility. Using (a6) :

$$U_j = \mu I_j/P - N_j^{\beta} \qquad (a15)$$

subject to the demand for its type of labor (a3) and the budget constraint :

$$I_j = W_j N_j + \sum_i V_{ij} + M_j \qquad (a16)$$

Solving this maximization problem gives :

$$\mu W_j/P = (\sigma/(\sigma-1))\beta N_j^{\beta-1}, \text{ or equivalently} \qquad (a17)$$

$$W_j/W = [((\sigma/(\sigma-1))(\beta/\mu)n^{1-\beta})(P/W)(N^{\beta-1})]^{\frac{1}{(1+\sigma(\beta-1))}} \qquad (a18)$$

Equation (a18) implies that the real wage, in terms of utility, is equal to $\sigma/(\sigma-1)$ times the marginal disutility of labor.

3. Market equilibrium

In equilibrium, desired real money balances must be equal to actual balances. Thus $M = M'$. Replacing in (a11) gives

$$Y = (\gamma/(1-\gamma))\, M/P \qquad (a19)$$

This is equation (5) in the text. Then, from equations (a4), (a9) and (a19), we get :

$$N = [n^{\frac{1}{1-\sigma}}(\gamma/(1-\gamma))^{\alpha} m^{-\alpha}]\left(\sum_{i=1}^{m}(P_i/P)^{-\alpha\theta}\right)(M/P)^{\alpha} \qquad (a20)$$

If all firms choose the same -not necessarily optimal- price, this reduces to :

$$N = [n^{\frac{1}{1-\sigma}} m^{1-\alpha} (\gamma/(1-\gamma))^{\alpha}]\,(M/P)^{\alpha} \qquad (a21)$$

Substituting equation (a19) into (a9) gives the demand function for product i, equation (7) in the text. Substituting equation (a21) into equation (a3) gives the demand function for labor of type j, equation (8) in the text. Note that as we have not used the price and wage rules to derive these demand functions, they hold even when prices or wages are not set optimally.

Substituting equation (a19) into (a14) gives the price rule for firm i, equation (10) in the text. Substituting (a19) into (a18) gives the wage rule for worker j, equation (11) in the text.

Bibliography

Akerlof, George, "Relative wages and the rate of inflation", QJE 83-3 (August 1969) : 353-374

Akerlof, George and Janet Yellen, "Can small deviations from rationality make significant differences to economic equilibria?", AER, 75-4 (September 1985,a) : 708-721

Akerlof, George and Janet Yellen, "A near-rational model of the business cycle, with wage and price inertia", forthcoming QJE, 1985,b

Blanchard, Olivier, "Price asynchronization and price level inertia", in "Indexation, Contracting and debt in an inflationary world", Dornbusch and Simonsen eds, MIT press, 1983 : 3-24

Blanchard, Olivier, "The wage price spiral", forthcoming QJE, 1985

Blanchard, Olivier and Stanley Fischer, "Macroeconomics", Chapters 8 and 9, mimeo 1985

Caplin, Andrew and Daniel Spulber, "Menu costs, inflation and endogenous relative price variability", mimeo, Harvard, 1985

Cooper, Russell and Andrew John, "Coordinating coordination failures in Keynesian models", mimeo, Cowles Foundation,745, April 1985

Diamond, Peter, "Aggregate demand management in search equilibrium", JPE, 90, October 1982 : 881-894

Dixit, Avinash and Joseph Stiglitz, "Monopolistic competition and optimum product diversity", AER, 67-3, June 1977, 297-308

Hart, Oliver, "A model of imperfect competition with Keynesian features", QJE 97-1 (February 1982) :109-138

Kiyotaki, Nobuhiro, "Macroeconomic implications of monopolistic competition", mimeo Harvard, April 1984

Mankiw, Gregory, "Small menu costs and large business cycles : a macroeconomic model of monopoly", QJE, 100-2 (May 1985) : 529-539

Mortensen, Dale, "The matching process as a non cooperative bargaining game", in "The economics of information and uncertainty", John McCall ed, University of Chicago Press and NBER, 1982

Solow, Robert, "Monopolistic competition and the multiplier", mimeo MIT, 1984

Startz, Richard, "Prelude to macroeconomics", AER, 74, (December 1984) : 881-892

Taylor, John, "Staggered price setting in a macro model", AER 69-2 (May 1979): 108-113

4353 U16

MIT LIBRARIES
3 9080 004 478 373

Date Due

APR 13 1990

MAY 03 1990

JAN. 04 1993

Lib-26-67

Bar Code
On Last
Page